Enrollment No: ______________________ **Exam Seat No:**______________________

C. U. SHAH UNIVERSITY

Winter Examination-2022

Subject Name : Complex Analysis

Subject Code : 5SC01COA1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 04/01/2023** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1		**Attempt the Following questions**	**(07)**
	a.	Solve $sinz = 2$.	**02**
	b.	Prove that $\cos h^2 x - \sin h^2 x = 1$.	**02**
	c.	Define entire function and give example of it.	**02**
	d.	Write C-R equation in polar form.	**01**
Q-2		**Attempt all questions**	**(14)**
	A	State and prove Cauchy Riemann equation.	**07**
	B	Find the product of all roots of $z^5 = 1 + i$.	**04**
	C	Find the value of $tan^{-1}(2i)$.	**03**
		OR	
Q-2		**Attempt all questions**	**(14)**
	A	Find the value of $(1+\sqrt{3}i)^{90} + (1-\sqrt{3}i)^{90}$.	**05**
	B	Find real and imaginary part of $\left[\frac{e}{2}(-1-\sqrt{3}i)\right]^{3\pi i}$.	**05**
	C	Prove that $sinh^{-1}z = log(z + \sqrt{z^2+1})$.	**04**
Q-3		**Attempt all questions**	**(14)**
	A	State and prove sufficient condition for a function to be analytic.	**05**
	B	Find harmonic conjugate of $\frac{y}{x^2+y^2}$.	**05**
	C	Show that $f(z) = xy + iy$is nowhere analytic.	**04**
		OR	
Q-3		**Attempt all questions**	
	A	State and prove C-R equation for analytic function	**06**
	B	Show that if $f(z)$ and $\overline{f(z)}$analytic in a given domain, then $f(z)$ must be constant throughout D.	**04**
	C	Show that $f(z) = \frac{1}{z^4} (z \neq 0)$ is differentiable in its domain.	**04**

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Find residue of $\frac{1}{z+z^2}$. **02**

b. Find the fixed points of transformation $w = \frac{6z-9}{z}$. **02**

c. Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{z^n (n!)^2}{2n!}$ **02**

d. State Liouville's Theorem. **01**

Q-5 **Attempt all questions** **(14)**

A State and prove Fundamental Theorem of Algebra. **05**

B Show that if $C$is the boundary of the triangle with vertices at the points $0, 3i$ and -4, oriented in the counterclockwise direction, then $\left|\int_C (e^z - \bar{z})\, dz\right| \leq 60$. **05**

C Integrate the function $f(z) = \frac{1}{z^4+4z^2}$ around the curve $C: |z - 2i| = 3$ traversed in counter-clockwise direction. **04**

OR

Q-5 **Attempt all questions**

A State and prove Cauchy's integral formula. **05**

B Find $\int_C f(z)dz$, where $f(z) = \pi exp(\pi\bar{z})$ and C is the boundary off the square with vertices at the points $0, 1, 1 + i$ and i, the orientation of $\boldsymbol{C}$ being in the counterclockwise direction **05**

C State and prove M-L inequality. **04**

Q-6 **Attempt all questions** **(14)**

A State and prove Laurent's series. **07**

B Find the Laurent expansions for the function $f(z) = \frac{z}{(z-2)(z+i)}$ in the regions $|z| > 2$. **04**

C Find the bilinear transformation that maps the points $2, i, -2$ in z-plane onto $1, i, -1$ in the w-plane. **03**

OR

Q-6 **Attempt all Questions**

A State and prove Taylor's series. **07**

B Find the Taylor series for the function $f(z) = \frac{1}{z}$ about the point $z_0 = 2$. **04**

C Evaluate $\int_C tanzdz$ $C: |z| = 2$ by using Cauchy's residue theorem. **03**